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Examining the Volatility of Ireland's Tax Base in the Paradigm of Modern Portfolio Theory

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Abstract

This paper extends a theoretical and empirical framework commonly applied in international finance to present an alternative paradigm within which issues of tax revenue volatility in Ireland can be studied. We establish a trade-off between revenue growth and volatility, typically associated with financial asset returns. We observe strong but time-varying cointegration among tax revenue streams. Statistical tests of mean-variance spanning suggest diversification benefits from holding Income Tax and Excise Duty. We also find that, from a mean-variance optimality perspective, the share of each tax in the State's tax portfolio as of 2017 is sub-optimal. Practical policy implications are discussed.

Keywords: Revenue Volatility, Portfolio Analysis, Fiscal Policy Making

JEL Classification: H2, G110, E610

Introduction

Prudential fiscal planning necessitates that long-term public spending be based on stable and less volatile sources of revenue. If a programme of expenditure is funded using windfall tax receipts, additional revenue (or spending cuts to other areas) will be required to continue to fund this programme should these windfall receipts fail to materialise in future years. In addition, revenue volatility complicates fiscal planning, as revenue forecasting is likely to be more challenging for more volatile taxes.

For ease of fiscal planning, and to ensure that public spending is linked to sustainable sources of revenue, a State might be interested in rebalancing the tax base and setting the share of each tax category at levels that minimise total tax revenue volatility. In other words, this would be the volatility-minimising tax base composition. Furthermore, an understanding of the "risk and return trade-off" for each of the State's main taxes more generally is valuable information for fiscal policy-makers to have.

Of course, the choice of the tax mix does not only concern the goal of minimizing volatility, but also includes wider issues such as equity and efficiency. Consideration should be given to how taxes impact on growth (some taxes are more distortionary than others), or influence the behaviour of tax-payers (whether intentional or otherwise). In effect, the optimal tax shares from the perspective of portfolio theory might differ substantially from the actual or desired tax shares when these wider issues are taken into account in designing the tax system (including feasibility constraints and social preferences). However, these issues have been well researched already (for a recent literature review of the evidence see Johansson (2016)) and are beyond the scope of our analysis. Instead, our paper aims to make a novel contribution by providing a theoretical discussion of how the State's portfolio of taxes might be structured if guided solely by the principles of risk and return.

This paper applies modern portfolio theory to present an alternative paradigm within which issues of volatility and growth in respect of State tax revenue can be studied. We compare the share of total tax revenue for each category of tax (Income Tax, Corporation Tax, etc.) to the financial asset shares in an investor's portfolio. In our analysis, we replace financial asset returns with the 'returns' of each tax category, that is; the revenue growth for each tax. Similarly, the volatility or risk of each tax category is assessed as the standard deviation of these 'returns' (a standard measure of volatility or risk in financial economics).

Generally the volatility of an investment portfolio is a function of the volatility of the underlying financial assets, their share in the investment portfolio and the covariance between these assets. Likewise, we model the overall volatility of total tax revenue as a function of:

1. the share of each tax category in the State's tax portfolio;
2. the volatility of each individual tax category; and
3. the covariance and correlation among these tax categories.

Intuitively, the higher the volatility of the individual revenue streams comprising the State's tax portfolio, and the more positive the correlation among these revenue streams (i.e. the extent to which they co-move); the greater total tax revenue volatility will be.

Some of our key findings are summarised below:

On the "risk-return trade-off":

- Our analysis highlights a trade-off between revenue growth and volatility; the most volatile taxes also experience the highest growth rate on average.

On tax revenue diversification:

- Using the Herfindahl–Hirschman Index, we establish that the volatility of the State's portfolio of taxes has increased in line with the level of diversification across the seven tax categories. This suggests that the State has diversified into more volatile taxes over time.
- We establish a strong degree of cointegration among the seven taxes, implying that there are limited benefits to diversifying across the primary tax categories. However, we find that the level of cointegration varies over time and has declined relative to the pre-crisis period.

On mean-variance optimality:

- We observe a statistically significant shift in the mean-variance efficient frontier from adding Income Tax and Excise Duty to a benchmark portfolio consisting of the six other taxes, while this result does not hold for each of the other tax categories.
- We find that the share of each tax in the State's tax portfolio as of 2017 is sub-optimal, from a mean-variance optimality perspective.
- We establish that the minimum variance portfolio consists of a majority holding of Excise Duty (65%), with some Income Tax (33%) and a relatively

small allocation for Customs (2%). We also observe some intertemporal instability in the variance-minimising shares of each tax, using a sub-period analysis.

Our results have some important policy implications. First, in designing tax policies that alter the composition of State tax revenue, consideration should be given to the volatility and growth of the overall tax portfolio (and the "risk-return trade-off" with respect to the taxes in that portfolio). Second, we suggest that a counter-cyclical fiscal fund has the potential to help mitigate the risks associated with over-weighting the tax portfolio with volatile revenue sources (a similar point is made in Addison-Smyth and McQuinn (2010)). This strategy could envisage setting aside a proportion of the exceeding difference between the current share of a volatile tax and its long-term average share (consider, for example, the temporarily large share of Stamp Duty during the Irish housing boom of the early-mid 2000s). Such funds would be used to stimulate the economy in a downturn and could serve to effectively smooth the business cycle.

This paper is divided into 6 main sections. Section 1 outlines the context to this research. In Section 2 we give an overview of the relevant literature, while in Section 3 we provide a preliminary analysis of the underlying data. Section 4 illustrates the statistical methods used in this paper and Section 5 details our empirical analysis. Finally, in Section 6 we conclude and discuss relevant policy implications.

1. Research Context

The risks associated with the volatility of Ireland's tax base (i.e. large yearly swings in government revenue), particularly in relation to Corporation Tax receipts, have come to the fore of debate on fiscal policy in Ireland. Having a tax base that consists of highly volatile revenue streams can potentially undermine the stability and sustainability of the public finances. Expenditure commitments of a permanent nature (e.g. public sector pay and pensions) should be linked to stable and less volatile revenue sources. If a new spending programme is funded using windfall tax receipts, additional revenue or spending cuts to other areas will be required to continue to fund this programme should these windfall receipts fail to materialise in future years. Furthermore, revenue volatility complicates fiscal planning, given that forecasting is likely to be more difficult for more volatile taxes. This paper investigates these issues and provides an analysis of the volatility of Ireland's tax base within the paradigm of modern portfolio theory.

In our view, modern portfolio theory can be generalised to an analysis of a state's tax revenue. In modern portfolio theory, which is drawn from financial economics, an investor has a portfolio made up of different financial assets, each of which has a different risk and expected return profile. The relationship between risk and return in relation to financial assets is intuitive; riskier investments will need to compensate would-be investors for taking on additional risk relative to alternative safer investment options. More generally, if a low-risk high-return asset did exist, rational investors would flock to that asset, driving up the price and reducing the rate of return to a level that the market deems more appropriate to that level of risk. The reverse would hold for a high-risk low-return asset.

If we consider a two-asset portfolio, the overall portfolio variance is calculated as follows:

$$V_p = ({w_a}^2{\sigma_a}^2) + ({w_b}^2{\sigma_b}^2) + 2(w_a\sigma_a w_b\sigma_b cov(a,b)) \qquad (1)$$

where w_a is the portfolio share of the first asset *a*; w_b is the portfolio share of the second asset *b*; σ_a is the standard deviation of asset *a*; σ_b is the standard deviation of asset *b*; and $cov(a,b)$ is the covariance between the two assets. The volatility of the portfolio is calculated as the standard deviation of the portfolio (the square root of the variance of the portfolio).

Generally, an investor would pursue two main investment strategies:

1. choosing the asset mix that minimises portfolio volatility for a given return; or
2. choosing the asset mix that maximises the portfolio return for a given level of volatility.

Those portfolios that represent the maximum return for a given level of risk (or the minimum level of risk for a given level of return) are said to lie on the mean-variance efficient frontier. These portfolios are mean-variance efficient, and there is no other portfolio that is more appealing within the investor's opportunity set.

Our empirical analysis is three-fold:

- We first examine the degree of correlation and cointegration (standard measures of the level of integration between financial market series) among the main tax categories comprising the State's total tax portfolio. Theoretically, the more integrated the revenue streams are, the smaller the diversification benefits from holding a portfolio consisting of these revenue streams.
- We then construct the mean-variance efficient frontier for a portfolio of all taxes (which begins with the global minimum variance portfolio). In practice, this involves minimising the portfolio variance equation in (1) subject to the constraint that the share of taxes in the portfolio be non-negative and sum to one, for various levels of return. We subsequently construct the efficient frontier for a portfolio of all taxes minus one, and compare this to the frontier consisting of the complete set of taxes. This enables us to visually examine the impact (if any) on the frontier from adding the missing category of tax, and we repeat this for each tax.
- Finally, we use statistical tests of mean variance spanning to assess if the change in the efficient frontier from adding the missing tax is statistically significant. This methodology allows us to examine the potential diversification benefits that each individual tax category brings to the State's portfolio of taxes, in terms of risk and return.

2. Literature

This section examines the relevant literature, and outlines how our paper complements and expands on this literature.

Relatively few papers have studied the volatility of tax revenue within the paradigm of modern portfolio theory. Existing studies tend to focus on the volatility of state taxes in the US, given the Federal governance system of the US and the extent to which fiscal policies are determined at state level. However, these studies do contain useful learnings that can be applied to the Irish context.

Crain (2003) is among the earliest papers to apply financial market theory to an analysis of tax revenue volatility at state level in the US, examining the trade-off between the growth rate of tax revenue and the volatility or riskiness of that revenue. This is commonly referred to as the risk-return trade-off in the context of financial markets. Our paper establishes the existence of a trade-off in the Irish context also, and further compares the risk and return profile of the primary taxes to investments in US Treasury Bills (T-Bills), the S&P 500, and an index of emerging market equities.

Matthews (2005) considers the funding of permanent expenditure using a volatile revenue source. The paper calculates the coefficient of variation (the ratio of the standard deviation and revenue growth) and prediction errors for property, sales and income taxes in the US state of Georgia. In particular, Matthews argues for the diversification of funding for state-wide education across multiple revenue sources, given the volatility inherent in a single tax. The paper further recommends building budgetary reserves in times of growth to preserve funding during economic downturns. We similarly examine the coefficient of variation as part of our analysis of the seven primary Irish taxes, and we also argue for placing more windfall revenue from volatile revenue sources into a counter-cyclical fiscal fund.

Garrett (2006) compares the actual revenue share of a tax to its variance minimising share, and examines how well a state's portfolio is constructed to minimise the variance in total tax revenue for the state. He argues that future work could examine the stability of variance minimising shares over different points of the business cycle, and proposes extending the model to allow for the analysis of multiple taxes rather than considering a specific tax against a combination of all "other" tax revenue. Our paper builds on this analysis by examining the variance minimising shares for multiple taxes (i.e. for all seven taxes in our sample), and across multiple time periods.

Cornia and Nelson (2010) conduct a similar assessment across US states, with each state representing a different portfolio of taxes. They construct the efficient frontier and examine how the actual composition of tax revenue across states compares with respect to the efficient frontier. From this, they argue that consideration should be given to the volatility and growth of a tax portfolio when designing tax policy changes that will impact on the composition of overall state revenue. We also examine how Ireland's tax portfolio compares to the efficient frontier, and further examine how the efficient frontier is impacted from the addition of each individual tax category, with both visual and statistical tests of mean-variance spanning.

McQuinn and Roche (2016) are the first to apply mean-variance analysis in the Irish (and indeed European) context. They examine Irish Exchequer receipts from 1984 to 2015 and estimate the efficient frontier based on *ex ante* tax forecasts and *ex post* outturns. Their analysis shows that an improvement of tax revenue growth relative to revenue volatility was possible if a larger proportion of tax revenue was derived from direct taxes (e.g. Income Tax) rather than indirect taxes (e.g. VAT). While we similarly analyse the efficient frontiers, we follow this visual examination with statistical tests of mean variance spanning. These tests allow us to assess the statistical significance of shifts in the efficient frontier from the addition of each individual tax to the benchmark portfolio consisting of the six other taxes.

There is a relatively larger body of literature in the wider area of tax elasticity and revenue buoyancy. These studies model revenue volatility as the elasticity of tax revenue with respect to changes in some macroeconomic variable, often GDP, and seek to determine the extent to which tax revenue moves with the business cycle. Pro-cyclical taxes tend to outperform expectations and experience strong outturns in times of growth, while the reverse is true during an economic downturn. As an example of this work in the Irish context, see Deli et al. (2016) and Acheson et al. (2017). As an alternative, our analysis focuses on the standard deviation of the revenue growth series as a measure of revenue volatility (as is standard practice in modern portfolio theory). Additional work has examined the implications of tax revenue volatility for revenue forecasting in Ireland (for example, Hannon (2014) analyses the source of forecast errors in an error decomposition framework).

3. Preliminary analysis

3.1. Data

We use annual tax revenue data taken from the Databank of the Department of Public Expenditure and Reform, which includes outturn data for the following tax categories:

- Income Tax;
- Valued Added Tax;
- Excise Duty;
- Corporation Tax;
- Stamp Duty;
- Customs; and
- Capital Taxes (Capital Gains Tax and Capital Acquisitions Tax).

The use of annual tax data means that we somewhat mitigate the issues relating to the seasonality of tax receipts (i.e. filing and consumption patterns). We exclude from our calculations the Training and Employment Levy, the Local Property Tax and Motor Vehicle Duties due to the short time series available. For consistency purposes, Capital Gains Tax and Capital Acquisitions Tax are aggregated and defined as Capital Taxes (these taxes have only been reported separately in outturn data since 2000). We use this revenue outturn data to calculate revenue growth rates for each tax and the standard deviation of these growth rates.

3.2. The revenue "risk and return trade-off"

Table 1 shows the mean return and standard deviation, as well as the ratio of the two, for the seven taxes in our sample from 1984 – 2017. We observe that Capital Taxes and Corporation Tax tend to have the highest average annual growth rates, and, alongside Stamp Duty, are among the most volatile in terms of standard deviation.

Among the largest taxes (Income Tax, VAT, Excise Duty and Corporation Tax) Corporation Tax has the highest standard deviation (17.3%), meaning that the annual percentage changes in Corporation Tax would tend on average to fluctuate significatly from their mean value (17.3 percentage points above or below the average annual change). Corporation Tax also exhibits the highest average annual growth rate at 12.2%, while Excise Duty grew on average by 4.2% each year over the period. Excise Duty is the tax category with the lowest standard deviation (5.6%). Income Tax, VAT, and Excise generally perform well in terms of their mean return or growth, relative to their volatility (0.96, 0.84 and 0.76 respectively).

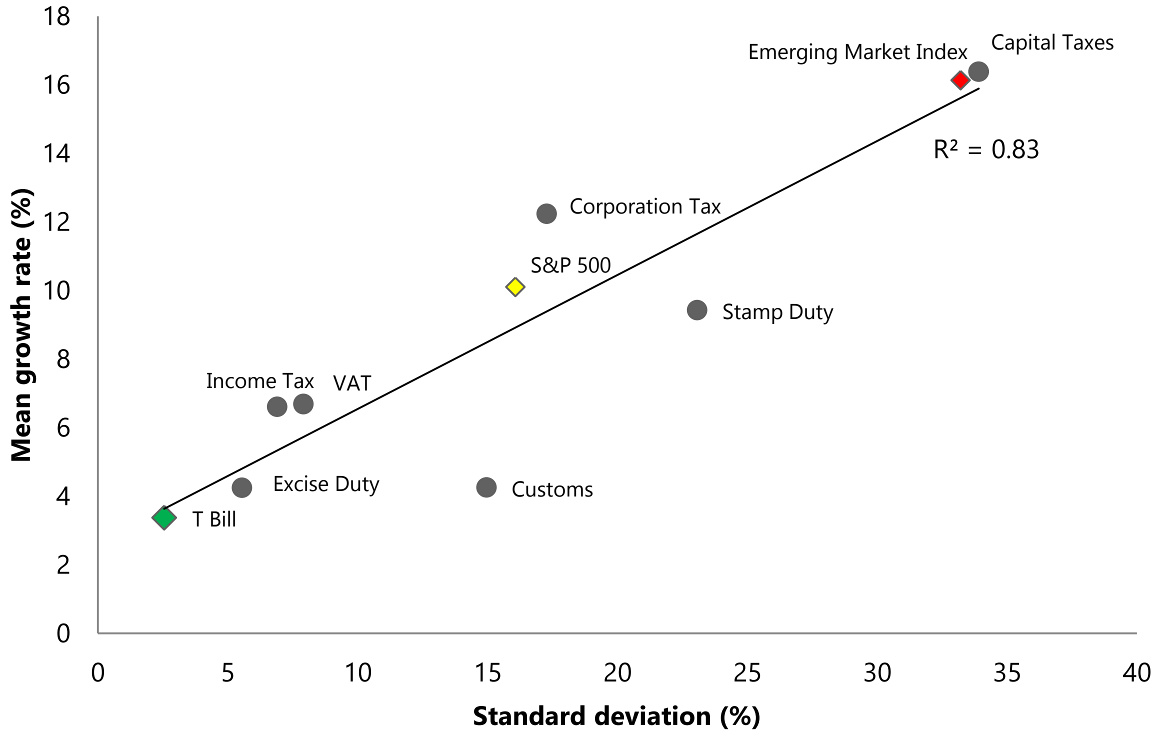
The data in Table 1 suggests the existence of a trade-off in terms of the rate of tax revenue growth (or return) and volatility, more typically associated with assets traded in financial markets. This trade-off is clearer from Figure 1, which for each tax category, plots the standard deviation against the average growth rate. In Figure 1, the risk-return trade-off is assessed relative to investments in US T-Bills (low volatility and low return investments), the S&P 500 (medium volatility and medium return investments), and an index of emerging market equities (high volatility and high return investments). Generally (with the exception of Corporation Tax), revenue streams having high volatility and large returns, such as Capital Taxes and Stamp Duty, are pro-cyclical, being based on transactions involving activities that are subject to 'boom and bust' cycles.

Table 1. Mean revenue growth and standard deviation, 1984 - 2017

Tax category	Mean growth rate (return)	Standard deviation (risk or volatility)	Return-risk ratio
Income Tax	6.61%	6.90%	0.96
VAT	6.68%	7.91%	0.84
Excise Duty	4.24%	5.55%	0.76
Corporation Tax	12.24%	17.27%	0.71
Stamp Duty	9.42%	23.07%	0.41
Customs	4.25%	14.96%	0.28
Capital Taxes	16.38%	33.91%	0.48

Notes: Capital Taxes comprise Capital Acquisitions Tax and Capital Gains Tax (reported separately in outturn data since 2000).

Figure 1. Risk-return trade-off among the main tax categories, 1984 - 2017

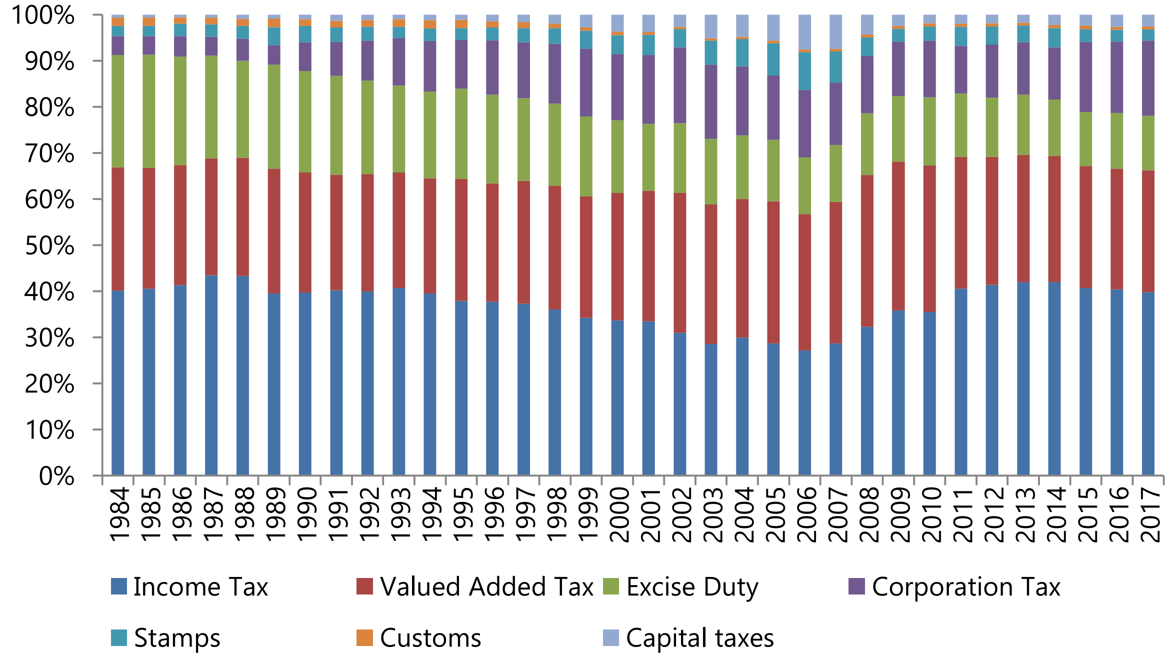


3.3. Ireland's tax revenue structure over time

As outlined previously, portfolio variance is a function of the volatility of the underlying assets, their share in the portfolio and the covariance between these assets. In this section, the shares are calculated in respect of total Exchequer Revenue, and therefore the total includes the Training and Employment Levy, the Local Property Tax and Motor Vehicle Duties, which are excluded from our sample elsewhere.

Figure 2 provides an overview of the trends in tax revenue shares from 1984 – 2017. Historically, Income Tax, VAT and Excise Duty have accounted for the largest proportion of total tax revenue (with a combined annual average from 1984 – 2017 of 80.2%). However, the share of Corporation Tax (16.2% in 2017) has recently surpassed the share of Excise Duty (11.7% in 2017), becoming the third most important revenue stream for the State. In recent years, Corporation Tax receipts have surged, increasing by 95% from 2012 to 2017, with a 49% increase in 2015 alone. This was largely influenced by the decision taken by foreign-owned multinationals to re-locate their assets and activities to Ireland. Corporation Tax revenue accounted for 16.2% of total tax revenue in 2017 (or €8.2 billion), which is well above the long term-average over 1984 – 2016 of 10.8%.

Figure 2. Trends in tax shares over time, 1984 – 2017



Source: Authors based on Department of Public Expenditure & Reform – Databank

The trend in Corporation Tax receipts emphasises the inherent volatility of this tax category, due to the responsiveness of receipts to wider economic conditions and to changes in tax regimes, both internationally and domestically. Corporation Tax receipts are also very concentrated, with data from the Revenue Commissioners (2018) highlighting that in 2017:

- the top 10 taxpayers accounted for 39% of receipts; and
- Foreign-owned multinationals accounted for 80% of receipts.

These findings highlight the extent to which Corporation Tax receipts are exposed to firm and sector-specific shocks.

The pre- and post-crisis period saw significant changes to Income Tax, which impacted on its relative share. Income Tax is the largest revenue stream of the Exchequer. This is true both historically and at present, with taxes on income (including USC) accounting for 39.4% (or €20 billion) of Exchequer revenue in 2017 (relative to an average of 36.4% over 1984 – 2016). The Income Tax share fell to a low of 27.2% in 2006. This is partially explained by the pre-crisis implementation of budget measures that narrowed the Income Tax base, and by the dramatic rise in VAT revenue during this time (Income Tax receipts grew by 36% from 2000 – 2006, while VAT receipts grew by 80% over the same period). In the aftermath of the fiscal crisis of 2008, in an attempt to restore the public finances, Income Taxes were increased and several tax expenditures were reduced. As a result, the Income Tax share peaked at around 42% in 2013 and 2014. In recent years, Income Tax receipts have grown strongly (increasing by 27% since 2013), notwithstanding the implementation of measures which have contributed to the narrowing of the base. In fact, Income Tax shows a significant degree of concentration with the top 13% of total Income Tax units (those with gross income above €70,000), accounting for 64% of total tax paid (including USC) in 2016.

Similarly to Income Tax, VAT receipts have increased by 28.7% since 2013. VAT is the second largest revenue stream accounting for 26.2% of total tax receipts (or €13.3 billion) in 2017, below the long-term average over 1984-2016 of 27.2%. However, the VAT share in total tax revenue has decreased in recent years, due to the increasing share of Corporation Tax.

During the boom period, the share of Stamp Duty in total tax revenue doubled from 4.1% in 2000 to 8.2% in 2006, before falling to 2.8% in 2009 (with a share of 2.4% in 2017). Similarly, for Capital Taxes, the share increased from 3.7% in 2000 to 7.6% in 2006, before falling to 2.4% in 2009 (with a share of 2.5% in 2017).

3.4. Diversification of the revenue base

Portfolio diversification has the potential to reduce portfolio volatility, assuming diversification across holdings that are not significantly positively correlated. Figure 3 shows the degree of concentration or diversification of tax revenue over time, alongside changes in the standard deviation of the State's tax portfolio. In particular, we are interested in assessing whether Ireland has diversified its tax portfolio over time, and how this compares with portfolio volatility.

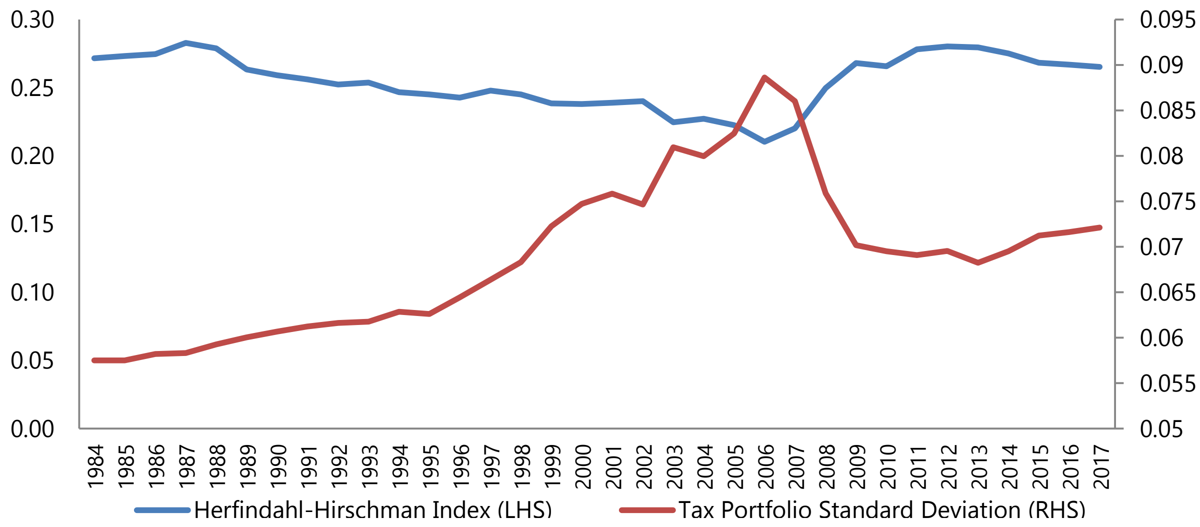
We calculate the level of concentration in the tax portfolio using the Herfindahl–Hirschman Index (HHI), which is an index typically employed in competition economics to measure the level of market power in an industry. While this approach is generally applied to market shares of firms, we extend it to an analysis of Ireland's tax shares. The index ranges between 0 and 1, with 1 being the case that all tax revenue is coming from a single source. Thus, the higher the index, the lower the degree of diversification of the tax base (i.e. the base is highly concentrated in a small number of taxes). The mathematical representation of the HHI index is as follows:

$$HHI = \sum_{i=1}^{N} w_i^2 \qquad (2)$$

where w_i is the relative tax share of a given tax category i, and N is the number of tax categories. Generally, financial market theory would suggest that more diversification is better in terms of risk reduction. However, this must take account of the co-movements among tax revenues and the underlying volatility of each revenue stream. The blue line in Figure 3 highlights that historically Ireland's tax base has been highly concentrated in a small number of taxes. From 1989 the tax base became more diversified, with the highest level of diversification reached in 2006. Since then, the trend has reversed and the tax base has become more concentrated.

The red line in Figure 3 highlights an interesting point: the increase in the diversification of tax revenue was associated with an overall increase in portfolio volatility or risk (we explore this in more detail in Section 5). The inverse relationship between the two series (with a correlation of -71%) was particularly evident during the boom period. This can be explained by the tax base shifting towards more volatile revenue sources (i.e. an increasing share of Capital Taxes and Stamp Duty). Addison-Smyth and McQuinn (2010, 2016) find that the period from 2002 to 2009 is characterised by substantial windfall gains in Stamp Duty and VAT, above levels warranted by underlying economic fundamentals. In this way, while tax revenue was seemingly more diversified during this time, this spike in the share of certain taxes was temporary and unsustainable. With the 2008 economic and financial crisis the inevitable collapse of these revenues, and the fiscal consolidation measures that followed, more emphasis was placed on relatively more stable sources of revenue, such as Income Tax.

Figure 3. Tax Portfolio Diversification and Volatility, 1984 – 2017



Notes: The HHI ranges from 0 to 1, with 1 being the case that all tax revenue is coming from a single source. As of end-2017, the tax base is largely concentrated around Income Tax, Excise Duty, Valued-Added Tax and Corporation Tax. These taxes make up 94% of the revenue aggregate across the taxes included in our sample.

4. Statistical Methods

4.1. Correlation and cointegration of tax revenue

Correlation

We examine the degree of linear association among the revenue streams of the seven tax categories in a bivariate framework using Pearson's correlation coefficient:

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}} \qquad (3)$$

The correlation coefficient is bound above by 1, and below by -1 (with 1 indicating a perfect positive correlation and -1 indicating a perfect negative correlation). When a pair of variables moves together in the same direction, there is a positive correlation between these variables. From an investment perspective, a portfolio consisting of underlying assets that exhibit strong positive co-movement (correlation) is undesirable, as an external shock will impact on the performance of all of the assets in the portfolio in a similar way. The same is true of a portfolio consisting of the State's tax revenues. Portfolio diversification generally involves allocating shares across entities that do not exhibit strong positive correlation with one another.

Cointegration

Following an analysis using Pearson's correlation coefficient, we model the long-term linkages among the seven tax categories as cointegrating relations using the Johansen-Juselius test (1990). This is a commonly used method in assessing the level of integration within a system or between a pair of variables, and has been employed extensively in studies of financial market integration (for example, see Quayes and Jamal (2016), Neaime (2016), and Babalos et al. (2016)). The Juselius-Johansen test is predicated on an assumption that the series contain a unit root, which we have pre-determined using an Augmented Dickey-Fuller test.

Essentially the Johansen-Juselius cointegration test involves calculating the test statistics for a vector auto-regressive (VAR) model in which each of our seven tax categories act as dependent variables. The resulting test statistics are used to identify the number of cointegrating vectors in the system defined by the VAR. The Johansen-Juselius test produces two test statistics. While the trace statistic examines the null hypothesis of *r* cointegrating vectors against the alternative hypothesis of *n* cointegrating vectors, the maximum eigenvalue statistic considers the null hypothesis of *r* cointegrating vectors against the alternative of *r + 1* cointegrating vectors. Johansen and Juselius (1990) provide asymptotic critical values.

We examine the level of cointegration in the complete set of taxes in a time-varying recursive framework, allowing for intertemporal changes in the extent to which tax revenue streams are integrated (following Hansen and Johansen (1993), Lucey and Voronkova (2005)). This recursive approach involves estimation over an initial period of size *t* (in our case this is the 16 years from 1985 to 2000) before expanding this period incrementally (by a single year) and re-estimating, and continuing until the end of the sample. We then plot each of the corresponding trace statistics, scaled to the 90 per cent critical value, which allows us to observe the dynamics of cointegration among the tax revenue streams over time.

4.2. Tests of mean-variance spanning

Investment theory has long advanced the view that investors should seek to diversify their holdings to reduce their risk exposure (see Markowitz, 1952). Modern portfolio theory is underpinned by the idea that the investment decision should be based not only on the risk-return profile of an individual asset, but on the relationship between that asset and the various other assets that the investor wants to hold.

Tests of mean-variance spanning (MVS) are used extensively in portfolio analysis to examine the implications of expanding a portfolio of assets, to include additional assets, in terms of the risk-return trade-off. Spanning tests were introduced by Huberman and Kandel (1987) and further developed by Kan and Zhou (2008), and have dominated the international finance literature since (see Berrill and Kearney (2008), Eiling et al. (2012), O'Hagan-Luff and Berrill (2015)).

MVS analysis considers a set of K "benchmark" and N "test" assets (typically financial assets), and investigates if; conditional on the set of K benchmark assets, the addition of N test assets can shift the mean-variance efficient frontier. In other words, MVS involves examining if the efficient frontier of the benchmark set coincides with the efficient frontier of the extended set ($K+N$). This amounts to examining if the set of benchmark assets yields the same diversification benefits as the extended set of benchmark (K) and test (N) assets.

In our analysis, the K benchmark assets are represented by six of the seven main tax categories, while the N test assets are represented by the seventh remaining tax (i.e. K = 6 and N=1). MVS analysis allows us to examine if the addition of the seventh tax to our portfolio of six taxes shifts the mean-variance efficient frontier. In other words, we test if including the seventh tax category in the portfolio of taxes improves the risk-return trade-off relative to the original benchmark of six taxes. We replicate this analysis with each of the seven tax categories acting as the test asset (*N*) in turn.

In practice, the typical spanning test involves the regression of the *N* test asset returns on the returns of the *K* benchmark assets as follows:

$$R_{N,t} = \alpha + \beta R_{K,t} + \varepsilon_t \quad (4)$$

with $\varepsilon_t \sim N(0, \sum)$, α = $\mathrm{E}[R_{N,t}] - \beta E[R_{K,t}] = \mu_N - \beta\mu_K$ and $\beta = V_{NK}V_{KK}^{-1}$. In the same way that the returns of financial assets represent the price growth of those assets, our analysis uses the 'returns' for each of the tax categories, or the revenue growth rate for each tax.

In defining $\delta = 1_N - \beta 1_K$, Huberman and Kandel (1987) and Kan and Zhou (2008) provide the necessary and sufficient conditions for spanning in terms of a restriction on α and δ such that we can test the following null hypotheses:

$$H^1_0\text{:}\ \alpha = 0_N, \quad H^2_0\text{:}\ \delta = 0_N \quad (5)$$

It follows that a test of whether the benchmark assets (*K*) span the extended set of benchmark plus test assets (*K+N*) involves jointly testing the above hypotheses. A failure to reject the null hypothesis implies that for the test asset (*N*), it is possible to form a portfolio of the benchmark assets (*K*), that has the same expected return (because $\alpha = 0_N$ and $\beta 1_K = 1_N$) but a lower variance (as $R_{K,t}$ and ε_t are uncorrelated while $Var(\varepsilon_t)$ is positive definite).

From an empirical standpoint, we use Ordinary Least Squares (OLS) estimation, and carry out a Wald test of the restrictions in (5). A detailed derivation of the statistical test of spanning (adapted from Kan and Zhou (2008) and Berrill and Kearney (2010)) is contained in the Appendix of this paper.

5. Empirical Analysis and Results

5.1. Correlation and cointegration of tax revenue

Modern portfolio theory advances that diversification benefits arise from holding a portfolio of underlying assets that do not display strong and positive linkages.

Table 2 displays bivariate correlation coefficients for all pairs of the seven tax categories. This shows the strength of the linear relationship between each pair of revenue streams. Overall, Customs is the least correlated with other taxes generally, while VAT appears to be the most correlated. The highest individual correlation is between VAT and Excise Duty (0.78), as expected, given that these taxes are levied on some of the same products. Similarly, Stamp Duty and Capital Taxes have the second highest correlation (0.67), reflective of the fact that they (like VAT and Excise Duty) share elements of the same tax base.

Table 2. Correlation matrix of tax categories

	Income	VAT	Excise	Corporation	Stamp	Customs	Capital
Income	*1.00*	0.47	0.34	0.37	0.46	0.27	0.46
VAT	0.47	*1.00*	0.78	0.47	0.59	0.31	0.60
Excise	0.34	0.78	*1.00*	0.44	0.54	0.19	0.50
Corporation	0.37	0.47	0.44	*1.00*	0.12	0.01	0.44
Stamp	0.46	0.59	0.54	0.12	*1.00*	0.22	0.67
Customs	0.27	0.31	0.19	0.01	0.22	*1.00*	0.15
Capital	0.46	0.60	0.50	0.44	0.67	0.15	*1.00*

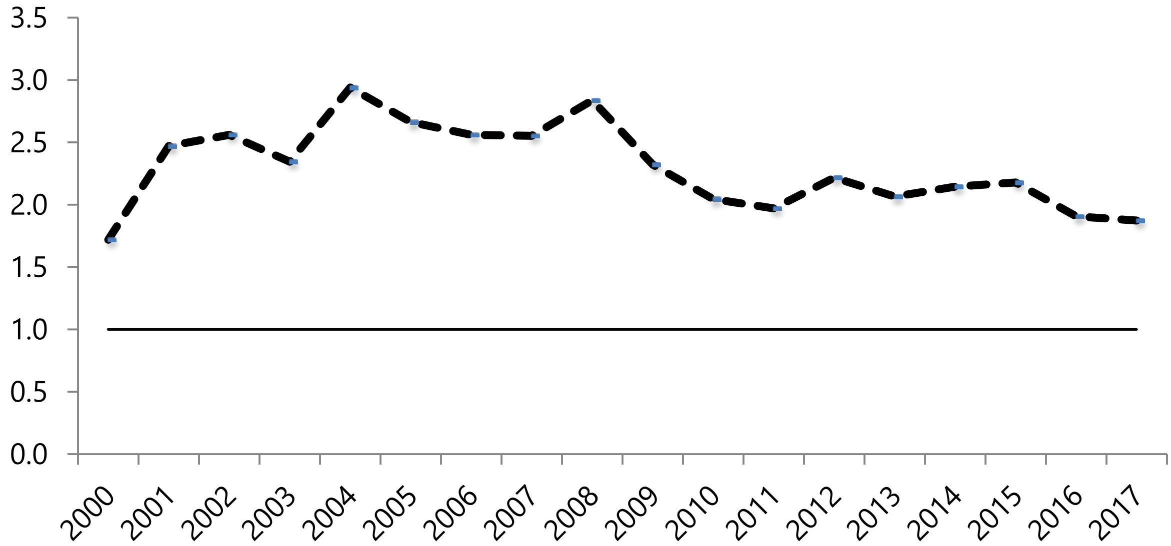
Notes: The correlation coefficient is bound above by 1 (indicating perfect positive correlation), and below by -1 (indicating perfect negative correlation).

We follow this correlation analysis by examining the long-term linkages among the seven tax categories. We model these linkages as cointegrating relations using the Johansen-Juselius test (1990). As previously discussed, this is a commonly used method in assessing the level of integration within a system or between a pair of variables, and has been employed extensively in studies of financial market integration (for example, see Quayes and Jamal (2016), Neaime (2016), and Babalos et al. (2016)).

Figure 4 shows the plot of the scaled trace statistic for each sub-sample in a recursive framework (scaled at the 90% significance level). When the graph trends above the horizontal line, there is cointegration among the tax revenue streams. As shown, there is considerable intertemporal instability in the level of cointegration among the tax revenues. We find particularly strong evidence of cointegration as the sample expands to include tax revenue from 2004 to 2008, suggesting a more integrated tax system in the years prior to the crisis. By the end of the sample, we observe relatively weaker evidence for cointegration, with a decline in the level of the scaled trace statistic between 2008 and 2011 (in line with fiscal consolidation efforts in the aftermath of the crisis) and between 2015 and 2017 (potentially reflecting the level shift in Corporation Tax due to multinationals relocating their assets to Ireland).

Generally, the level of cointegration remains substantially below pre-crisis levels, suggesting that, while there remain substantial linkages between the tax revenue streams, these linkages are weaker than in the pre-crisis period, and could be explained by efforts to broaden the tax base in response to the fiscal crisis. This merits further analysis and consideration in future work.

Figure 4. Plot of the scaled trace statistics



Notes: The dashed line shows the scaled trace statistic (at the 90 per cent significance level). When the dashed line is above the solid horizontal line, there is cointegration in the system. The 90 per cent critical value is 118.5. The initial window runs from 1984 to 2000, the sample is then expanded incrementally by a single year. This process is repeated until the end of the sample is reached (in 2017), at which point the analysis is over the entire sample (from 1984-2017).

5.2. Estimating optimal revenue shares

In this section we estimate optimal revenue shares as those that comprise a portfolio that lies on the mean-variance efficient frontier. Generally, the efficient frontier displays the set of portfolios that offer the highest expected return for a given level of risk, or the lowest risk for a given level of expected return. Portfolios that lie beyond the frontier are unattainable, and are not part of the investor's investment opportunity set, while those that lie below the frontier are sub-optimal, as it is possible to select an alternative portfolio that offers a higher expected return for the same level of risk.

In constructing the efficient frontier, we begin by establishing the global minimum variance portfolio, that is, the combination of shares that achieves the lowest possible level of risk (the minimum variance portfolio marks the beginning of the frontier). This involves optimising (minimising) the portfolio variance equation in (1) subject to the constraint that the share of taxes in the portfolio be non-negative and sum to one.

In doing so, we establish the variance minimising shares detailed in Column 2 of Table 3. As shown, the variance minimising portfolio of taxes would include a majority share of Excise Duty (65%) followed by Income Tax (33%) with some additional revenue from Customs (2%). Although the inclusion of Customs appears unusual from a risk-return perspective, this potentially reflects the relatively low level of correlation between Customs and each other tax. These shares differ substantially from the historical average for the sample, and from the 2017 shares, also detailed in Table 3.

The global minimum variance portfolio shows how the tax system would be structured if policy-makers were motivated solely by the goal of minimising volatility. However, targeting the global minimum variance portfolio of tax revenue may not be desirable or feasible from a policy-maker's perspective. Specifically, the underlying base may not generate sufficient revenue, and there are important issues of progressivity and equity that may make a particular allocation undesirable. Furthermore, governments may not have the political capital or will to make substantial changes to the current tax system. We consider this first simulation as a theoretical benchmark exercise.

For a more realistic outcome, we replicate this analysis imposing minimum and maximum "feasibility constraints" i.e. we impose minimum and maximum limits on what each share can be, that are intended to more accurately represent the limitations on the opportunity set of policy-makers. These feasibility constraints represent the sample minimum and maximum shares for each tax, and are as shown in Columns 5 and 6 of Table 3. The variance minimising shares '*With Bounds*' are shown in the last Column. Income Tax, Excise and Customs converge to their sample maximum values, while VAT, Corporation Tax and Capital Taxes converge to their sample minimum.

Garrett (2006) states that, as the variance minimising shares are a function of the sample period used, the stability of these estimates should be examined over multiple periods of the business cycle. In effect, this involves establishing the minimum variance portfolio for each sub-period (absent any feasibility constraints), and noting the optimal tax shares in each case. We examine the sub-period from 1985 to 1999, and also examine pre-crisis (2000 - 2007) and post-crisis (2008 – 2017) sub-periods. These results are shown in Table 4. We find that the variance minimising share of Income Tax has been declining over time, from 32% between 1985 to 1999, to 20% in the pre-crisis period, and 16% in the post-crisis period. Conversely, the variance minimising share of Excise Duty has increased from 41% pre-crisis, to 84% post-crisis. While VAT and Corporation Tax have variance minimising shares of 30% and 10% pre-crisis respectively, these shares drop to 0% post-crisis. In optimally allocating shares to minimise portfolio variance, it appears that there has been a move away from a more diversified tax portfolio, towards greater concentration on less volatile taxes (as also observed in Figure 3).

Table 3. Variance minimising versus actual shares for each tax category

Tax Category	**Variance Minimising**	**Actual average (1985 – 2017)**	**2017**	**Sample Min.**	**Sample Max.**	**With Bounds**
Income Tax	33%	37%	40%	27%	44%	42%
VAT	0%	28%	27%	25%	33%	25%
Excise Duty	65%	17%	12%	12%	25%	25%
Corporation Tax	0%	11%	16%	4%	16%	4%
Stamp Duty	0%	4%	2%	2%	8%	2%
Customs	2%	1%	1%	0%	2%	2%
Capital Taxes	0%	2%	3%	1%	8%	1%

Notes. The above figures are rounded to the nearest percentage, which may affect totals.

Table 4. Variance minimising shares for each tax category, pre and post-crisis

Tax Category	Full Sample 1985 - 2017	1985 - 1999	Pre-Crisis 2000 - 2007	Post-Crisis 2008 - 2017
Income Tax	33%	32%	20%	16%
VAT	0%	0%	30%	0%
Excise Duty	65%	56%	41%	84%
Corporation Tax	0%	3%	10%	0%
Stamp Duty	0%	0%	0%	0%
Customs	2%	9%	0%	0%
Capital Taxes	0%	0%	0%	0%

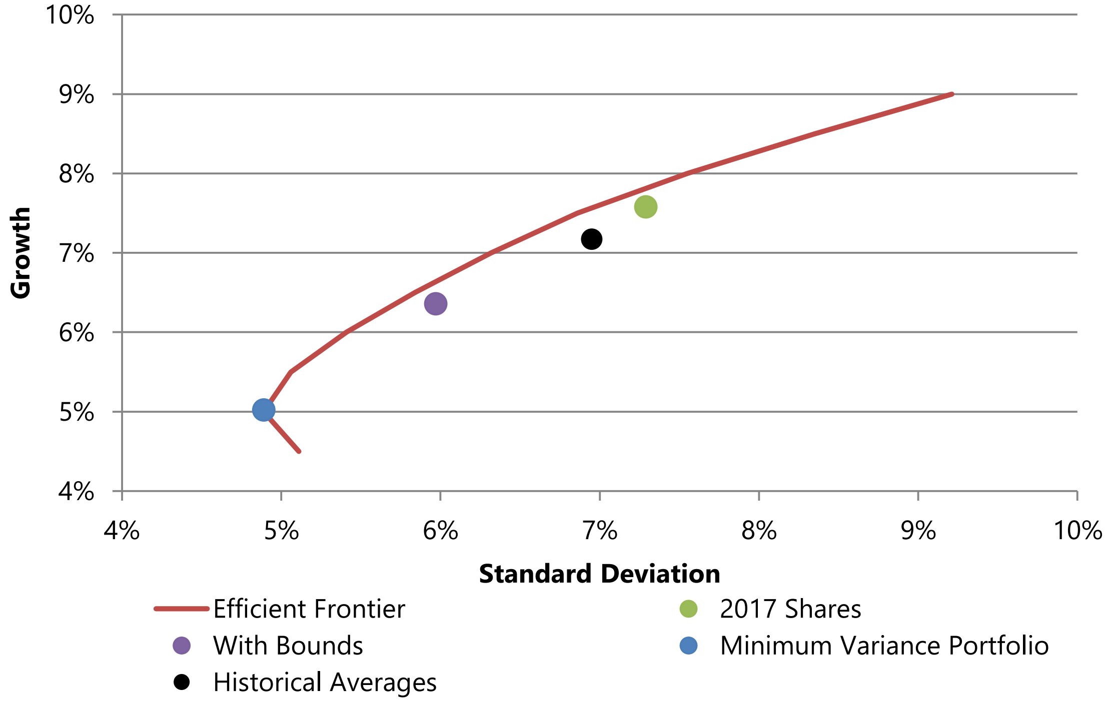
Notes. The above figures are rounded to the nearest percentage which may affect totals.

Having established the global minimum variance portfolio, we proceed by constructing the efficient frontier. We optimise equation (1) for different levels of risk/volatility, and plot the results in risk-return space as shown in Figure 5 below. We also plot each of the portfolio allocations detailed in Table 3, that is:

- the global minimum variance portfolio;
- the minimum variance portfolio with the sample minimum and maximum share bounds;
- the portfolio consisting of sample average shares; and,
- the portfolio consisting of 2017 shares.

As shown, with the exception of the global minimum variance portfolio (which by definition lies on the efficient frontier), all of the plotted portfolios lie below the frontier. This indicates that, in risk-return terms, these are sub-optimal portfolios, and the tax shares could be reallocated in a way that improves the level of expected return for the inherent level of risk in each case.

Figure 5. Efficient frontier of the complete set of taxes, 1985 - 2017



5.3. Mean-variance spanning

In the preceding section, we established the efficient frontier that indicates the optimal portfolio allocations for the complete set of seven taxes. We proceed with a mean-variance spanning analysis, that is; for each possible combination of six of the seven tax categories, we assess if the addition of the seventh tax shifts the mean-variance efficient frontier. In effect, this informs us whether the addition of the seventh tax in each case presents diversification benefits (i.e. improvements in the risk-return trade-off).

Figures 6a to 6f contain the efficient frontiers for each combination of six taxes. As shown, there is a visual improvement in the efficient frontier from adding Income Taxes to a portfolio consisting of the six other tax categories, with a visible frontier shift. We observe only minor improvements from including Corporation Tax, VAT and Excise Duty, while the frontiers with and without Stamp Duty and Customs, appear to overlap (or 'span').

While this graphical analysis provides a useful and intuitive interpretation of the benefits from adding the additional tax in each case, we proceed with a test of the statistical significance of these shifts in the efficient frontiers.

Figure 6a. Income Tax

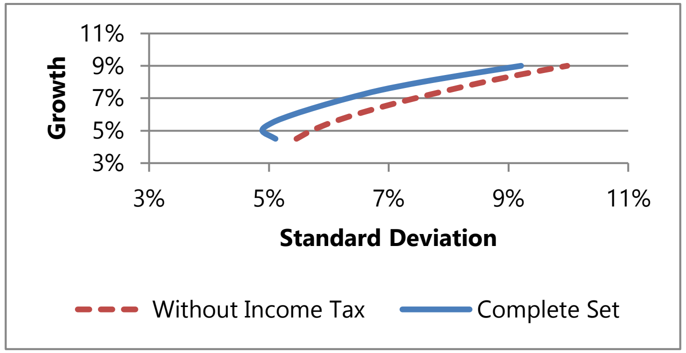


Figure 6b. VAT

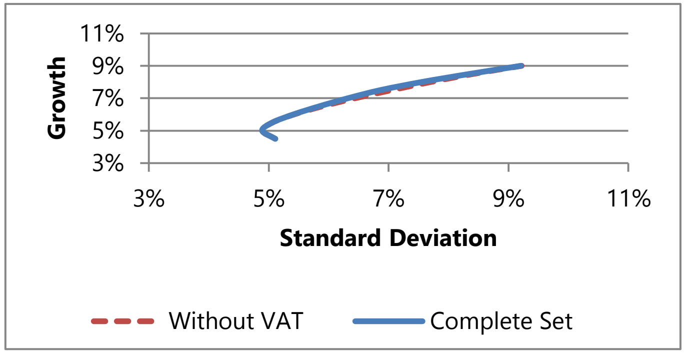


Figure 6c. Corporation Tax

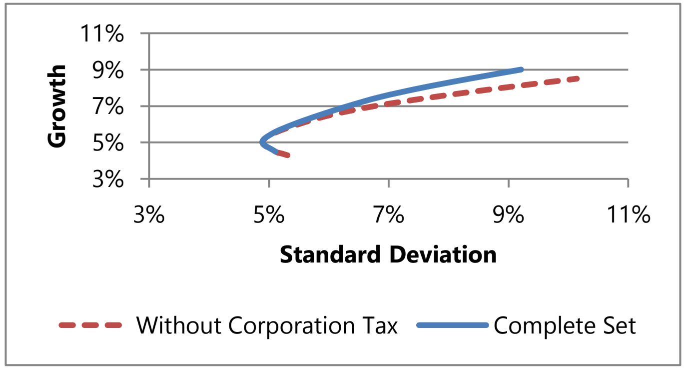


Figure 6d. Excise Duty

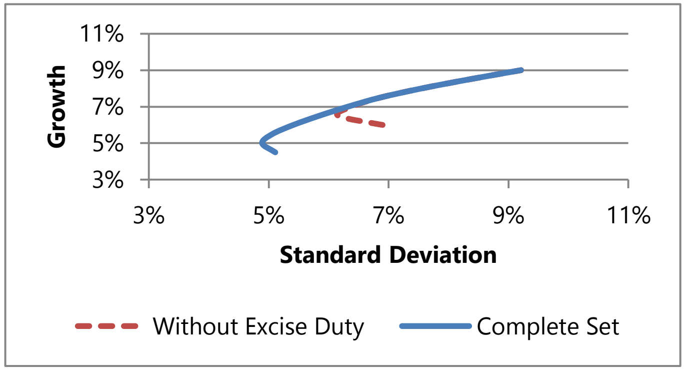


Figure 6e. Stamp Duty

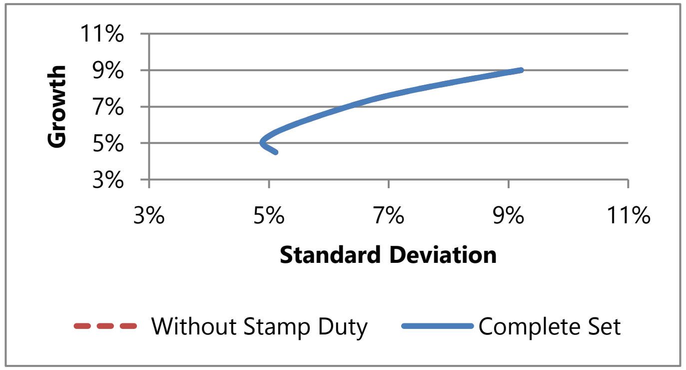
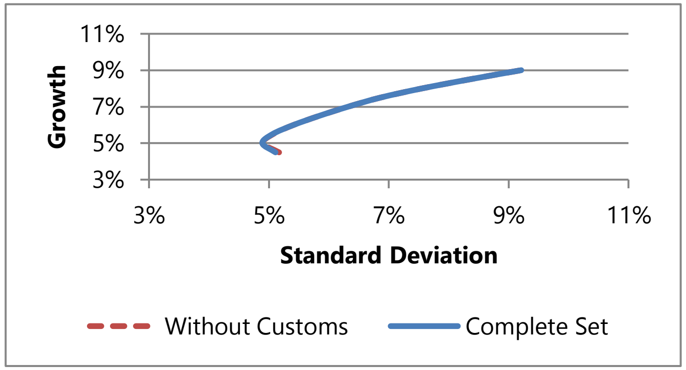


Figure 6f. Customs



5.4. Statistical tests of spanning

The results of our statistical tests of mean-variance spanning are shown in Table 5. Intuitively, the lower the *p* value (in parentheses), the higher the expected return per unit of risk from adding the additional tax category to the State's portfolio of taxes (or the lower the risk per unit of return). As shown, we can reject the null hypothesis of spanning at the one per cent level for both Income Tax and Excise Duty. Consistent with our graphical analysis of the efficient frontiers in Figures 6a and 6d above, this suggests that there are benefits in terms of the risk-return trade-off from including Income Tax and Excise Duty, to a portfolio consisting of the six other taxes.

However, this result does not hold for the rest of the tax categories, that is, there are no statistically significant benefits in terms of the risk-return trade-off, from adding VAT, Corporation Tax, Stamp Duty, Customs or Capital Taxes to a portfolio of the six other taxes. This is a somewhat surprising result, particularly as VAT offers a better risk-return trade-off (i.e. a higher ratio of growth to standard deviation) than Excise Duty. However, this could be explained by the relatively high correlation between the two (at 0.78, see Table 2) and the lower correlation between Excise Duty and each other tax category generally, relative to VAT (lower correlation allows for potentially greater diversification benefits). In addition, VAT offers a similar average growth rate to Income Tax (6.7% versus 6.6% respectively) but with a higher level of risk or volatility (7.9% to 6.9% respectively).

We can conclude that there is only a statistically significant shift in the mean-variance efficient frontier from adding holdings of Income Tax or Excise Duty, to a State portfolio consisting of the six remaining tax categories. Whereas, for VAT, Corporation Tax, Stamp Duty, Customs and Capital Taxes, it is possible to form a tax portfolio of the six remaining taxes that has the same expected return but a lower volatility.

Table 5. Mean variance spanning results

Test Assets	Test Statistic (*p* value)
Income Tax	6.39***
	(0.006)
VAT	0.70
	(0.504)
Excise Duty	14.40***
	(0.000)
Corporation Tax	1.86
	(0.175)
Stamp Duty	1.35
	(0.278)
Customs	0.10
	(0.908)
Capital Taxes	0.32
	(0.726)

Notes: In each case, the null hypothesis (H_0) is that the benchmark assets span the extended set of benchmark plus test assets. The test assets are as indicated in each case, while the benchmark assets are the remaining six tax categories. ******* Indicates statistical significance at the one per cent level. Capital Taxes refers to both Capital Acquisitions Tax (CAT) and Capital Gains Tax (CGT). We combine these two for the purpose of our MVS analysis to ensure a lengthy and consistent sample (outturns for CGT and CAT were reported jointly under Capital Taxes until 2000).

Conclusion

This paper presents several key empirical findings. We establish that a trade-off exists between tax revenue growth and volatility, more typically associated with the returns of financial assets. From a diversification perspective, we find that the overall volatility of the State's tax portfolio has increased in line with the level of diversification across the seven main tax categories. We further establish a strong degree of cointegration among the tax revenue streams, implying that there are minimal diversification benefits. However, our analysis establishes that the level of cointegration varies substantially over time, and has declined relative to the pre-crisis period. Finally, in terms of mean-variance spanning, we observe a statistically significant shift in the efficient frontier from adding Income Tax and Excise Duty to a benchmark portfolio consisting of the six remaining taxes, while this result does not hold for each of the other taxes. We also find that the share of each tax in the State's tax portfolio as of 2017 is sub-optimal, from a mean-variance optimality perspective.

Our paper presents optimal tax revenue shares from the perspective of modern portfolio theory. The purpose of this analysis is to estimate the tax shares which minimise tax portfolio volatility. However, the resulting shares may be undesirable or infeasible from the policy-maker's perspective, when also considering issues of equity and efficiency. This paper aims to make a novel contribution by providing a theoretical discussion of how the State's portfolio of taxes might be structured if guided solely by the principles of risk and return.

However, our results also have a number of practical policy implications. In the design of tax policy, greater consideration should be given to the ways in which a policy change could materially alter the composition of the tax revenue of the State. Specifically, consideration should be given to the growth and volatility of the overall tax portfolio in designing tax policy.

Furthermore, the use of counter-cyclical fiscal funds should be considered as a potential mitigant to the risks associated with an over-reliance on more volatile revenue streams. This could involve using a proportion of the excess revenues from a volatile tax head to capitalise a counter-cyclical fund that would then be used to fund expenditure when that volatile revenue fails to materialise in future years. We have seen evidence of this in the Irish context, with the National Training Fund (sourced by a levy on employers and used to fund certain further education and training programmes) and the Social Insurance Fund effectively acting as counter-cyclical funds. While payments into the Funds are inherently pro-cyclical (larger in times of high employment), surpluses generated in periods of strong economic performance allowed for a relatively stable source of funding throughout the fiscal crisis. With the stated goal of implementing counter-cyclical fiscal policy, the Irish Government is currently establishing a 'Rainy Day Fund' (RDF). RDFs are quite common in US States, but these are less utilised in European countries.

Future work in this area could examine statistical tests of mean-variance spanning in a sub-period framework, across different time periods and at different points of the business cycle. Furthermore, a robustness test of our results might involve removing the discretionary component from tax revenue volatility (i.e. controlling for the impact of policy change) prior to analysis. This could build upon previous research in the area which has implemented the proportional adjustment method to disentangle the impact of discretionary measures (for reference, see Princen et al (2013) and Barrios and Fargnoli (2010) for an analysis of discretionary tax measures in the European Union; and Casey and Hannon (2016) for an analysis of Corporation Tax in an Irish context). Finally, future work should seek to explore the origins of the risk-return trade-off identified in our paper, and the drivers of revenue volatility for each of the seven taxes. This in turn would make a valuable contribution in terms of the accuracy of revenue forecasts, and would aid in fiscal planning.

References

Acheson, J., Yota, D. D., Lambert, D., and E. L. W. Morgenroth, 2017. "Income Tax Revenue Elasticities in Ireland: An Analytic Approach", *Research Series*, No. 59, Economic and Social Research Institute.

Addison-Smyth, D., and K. McQuinn, 2016. "Assessing the Sustainable Nature of Housing-Related Taxation Receipts: The Case of Ireland", *Journal of European Real Estate Research,* June.

Addison-Smyth, D., and K. McQuinn, 2010. "Quantifying Revenue Windfalls from the Irish Housing Market", *Economic and Social Review*, Vol. 41(2), pp. 201-223.

Babalos, V., Caporale, G. M., and N. Spagnolo, 2016. "Equity Fund Flows and Stock Market Returns in the US Before and After the Global Financial Crisis: A VAR-GARCH-In-Mean Analysis", *DIW Berlin Discussion Paper*, No. 1583.

Barrios, S., and R. Fargnoli, 2010. "Discretionary Measures and Tax Revenues in the Run-up to the Financial Crisis", *European Economy Economic Papers*, 419.

Berrill, J., and C. Kearney, 2010. "Firm-Level Internationalisation and the Home Bias Puzzle", *Journal of Economics and Business*, Vol. 62, No. 4.

Casey, E. and A. Hannon, 2016. "Challenges Forecasting Irish Corporation Tax", *Analytical Note No. 10*, Irish Fiscal Advisory Council.

Cornia, G. C. and R. D. Nelson, 2010. "State Tax Revenue Growth and Volatility", *Regional Economic Development*, Vol. 6, No. 1, pp. 23-58, Federal Reserve Bank of St. Louis.

Crain, M. W., 2003. *Volatile States: Institutions, Policy, and the Performance of American State Economies*, University of Michigan Press.

Deli, D. Y., Rodriguez, A. G., Kostarakos, I., and P. Varthalitis, 2018. "Dynamic Tax Revenue Buoyancy Estimates for a Panel of OECD Countries", *ESRI Working Paper*, No. 592, Economic and Social Research Institute.

Eiling, E., Gerard, B., Hillion, P., and F. A. de Roon, 2012. "International Portfolio Diversification: Currency, Industry and Country Effects Revisited", *Journal of International Money and Finance*, Vol. 31, No. 5, pp. 1249-1278.

Garrett, T. A., 2006. "Evaluating State Tax Revenue Variability: A Portfolio Approach", *Working Paper Series*, 2006-008A, Federal Reserve Bank of St. Louis.

Hannon, A., 2014. “Tax Forecasting Error Decomposition”, *Analytical Note*, No. 3, Irish Fiscal Advisory Council.

Hansen, H. and S. Johansen, 1993. “Recursive Estimation in Cointegrated VAR-Models”, *Discussion Papers*, No. 92-13, University of Copenhagen Department of Economics.

Huberman, G., and S. Kandel, 1987. “Mean-variance Spanning”, *The Journal of Finance*, Vol. 42, No. 4.

Jobson, J. D., and B. Korkie, 1989. “A Performance Interpretation of Multivariate Tests of Asset Set Intersection, Spanning, and Mean-Variance Efficiency”, *Journal of Financial and Quantitative Analysis*, Vol. 24, No. 2, pp. 185-204.

Johansen, S., and K. Juselius, 1990. “Maximum Likelihood Estimation and Inference on Cointegration – With Applications to the Demand for Money”, *Oxford Bulletin of Economics and Statistics*, Vol. 52, No. 2.

Johansson, Å., 2016. "Public Finance, Economic Growth and Inequality: A Survey of the Evidence", *OECD Economics Department Working Papers*, No. 1346, OECD Publishing, Paris, https://doi.org/10.1787/094bdaa5-en.

Kan, R., and G. Zhou, 2008. “Tests of Mean-Variance Spanning”, *OLIN Working Paper*, No. 99-05.

Lucey, B. M., and S. Voronkova, 2005. “Russian Equity Market Linkages Before and After the 1998 Crisis: Evidence from Time-Varying and Stochastic Cointegration Tests”, *BOFIT Discussion Paper*, 12/2005.

Markowitz, H., 1952. “Portfolio Selection”, *The Journal of Finance*, Vol. 7, No. 1.

Matthews, J., 2005. “Tax Revenue Volatility and a State-wide Education Sales Tax”, *Fiscal Research Centre*, Policy Brief Number 109, Andrew Young School of Policy Studies.

McQuinn, K., and J. M. Roche, 2016. “Efficient Frontiers and Fiscal Stability: An Ex-ante and Ex-post Application to the Irish Public Finances”, *ESRI Working Paper*, No. 538, Economic and Social Research Institute.

Neaime, S., 2016. “Financial Crises and Contagion Vulnerability of MENA Stock Markets”, *Emerging Markets Review*, Vol. 27, pp. 14-35.

O' Hagan-Luff, M., and J. Berrill, 2015. "Why Stay-at-Home Investing Makes Sense", *International Review of Financial Analysis*, Vol. 38, pp. 1-14.

Princen S., Mourre G., Paternoster P., and G-M. Isbasoiu, 2013. "Discretionary Tax Measures: Pattern and Impact on Tax Elasticities", *European Economy Economic Papers*, 499.

Quayes, S., and A. M. M. Jamal, 2016. "Impact of Demographic Change on Stock Prices", *The Quarterly Review of Economics and Finance*, Vol. 60, pp. 172-179.

Revenue Commissioners (2018). Corporation Tax 2017 Payments and 2016 Returns.

Appendix: Derivation of the spanning tests

Tests of spanning can be derived as follows (adapted from Kan and Zhou (2008) and Berrill and Kearney (2010)). Consider the following model in matrix notation (note that this is simply equation (4) in matrix form):

$$R = X\beta + \sum , \qquad (6)$$

The unconstrained maximum likelihood estimates of β and ∑ are determined by

$$\hat{\beta} = (X'X)^{-1}(X'R) \text{ and, } \hat{\sum} = \frac{1}{T}(R - X\hat{\beta})'(R - X\hat{\beta}) , \qquad (7)$$

The spanning test can be derived by defining $\hat{\mu} = \sum_{t=1}^{T} \frac{R_t}{T}$ and $\hat{V} = \frac{1}{T}\sum_{t=1}^{T}(R_t - \hat{\mu})(R_t - \hat{\mu})'$, and four constants *a, b, c* and *d*.

Consider the two efficient frontiers: the frontier for *K* assets, and the frontier for *K+N* assets. Firstly, for the *K* assets:

$$\hat{a}_K = \hat{\mu}'_K \hat{V}^{-1}{}_{11} \hat{\mu}_K \qquad \hat{b}_K = \hat{\mu}'_K \hat{V}^{-1}{}_{11} 1_K \qquad \hat{c}_K = \hat{1}'_K \hat{V}^{-1}{}_{11} 1_K \qquad \hat{d}_K = \hat{a}_K \hat{c}_K - \hat{b}^2{}_K$$

While for the *K+N* assets:

$$\hat{a}_{K+N} = \hat{\mu}'_{K+N} \hat{V}^{-1} \hat{\mu}_{K+N} \qquad \hat{b}_{K+N} = \hat{\mu}'_{K+N} \hat{V}^{-1} 1_{K+N}$$

$$\hat{c}_{K+N} = \hat{1}'_{K+N} \hat{V}^{-1} 1_{K+N} \qquad \hat{d}_{K+N} = \hat{a}_{K+N} \hat{c}_{K+N} - \hat{b}^2{}_{K+N}$$

In moving to the frontier of the extended set of assets, *K+N,* from the frontier of the benchmark assets, *K,* the above constants change by:

$$\Delta\hat{a} = \hat{a}_{K+N} - \hat{a}_K \qquad \Delta\hat{b} = \hat{b}_{K+N} - \hat{b}_K \qquad \Delta\hat{c} = \hat{c}_{K+N} - \hat{c}_K$$

We can then form the following matrices:

$$\hat{G} = \begin{vmatrix} 1 + \widehat{a_K} & \widehat{b_K} \\ \widehat{b_K} & \hat{c}_K \end{vmatrix} \text{ and, } \hat{H} = \begin{vmatrix} \Delta\hat{a} & \Delta\hat{b} \\ \Delta\hat{b} & \Delta\hat{c} \end{vmatrix} \qquad (8)$$

Combining these two matrices, allowing $\hat{\Sigma}$ to denote the unconstrained maximum likelihood estimate of Σ with *K+N* assets in (7), and allowing $\tilde{\Sigma}$ to denote the constrained maximum likelihood estimate of Σ with *K* assets in (7), and defining $U = |\hat{\Sigma} \quad \widetilde{\Sigma^{-1}}|$, then the likelihood ratio test of whether the *K* benchmark assets span the extended set of *K+N* benchmark and test assets is given by:

$$LR = -T\ ln\ (U) \qquad (9)$$

where,

$$U = |\hat{\Sigma} \quad \widetilde{\Sigma^{-1}}| = \frac{|\hat{G}|}{|\hat{G}+\hat{H}|} = \frac{(1+\widehat{a_K})\hat{c}_K - \hat{b}^2{}_K}{(1+\hat{a}_{K+N})\hat{c}_{K+N} - \hat{b}^{\,2}{}_{K+N}} = \left(\frac{\hat{c}_K}{\hat{c}_{K+N}}\right)\left(\frac{1+\frac{\hat{d}_K}{\hat{c}_K}}{1+\frac{\hat{d}_{K+N}}{\hat{c}_{K+N}}}\right) \qquad (10)$$

Under the null hypothesis, the distribution of the likelihood ratio test is:

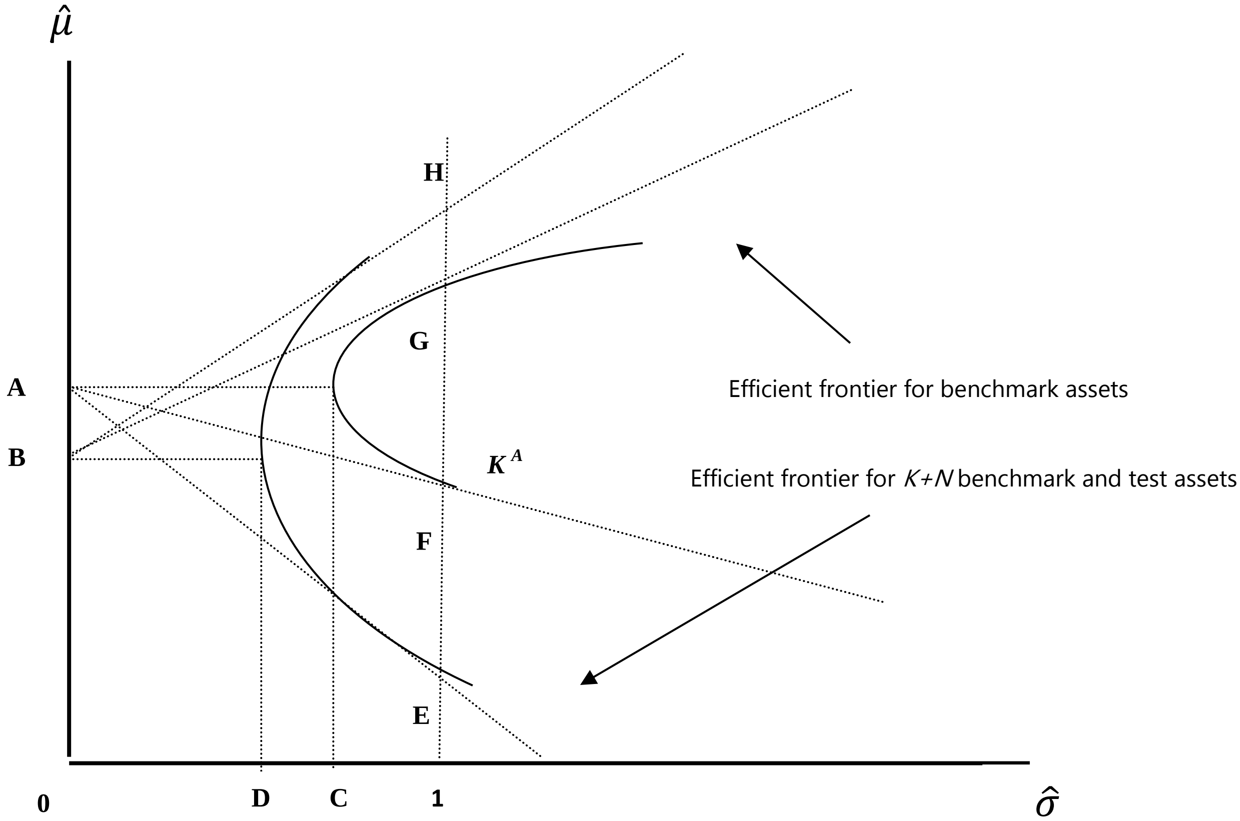
$$F = \left(\frac{T-K-N}{N}\right)\left(U^{-\frac{1}{2}} - 1\right) = \left(\frac{T-K-N}{N}\right)\left[\left(\frac{\sqrt{\hat{c}_{K+N}}}{\sqrt{\hat{c}_K}}\right)\left(\frac{\sqrt{1+\frac{\hat{d}_{K+N}}{\hat{c}_{K+N}}}}{\sqrt{1+\frac{\hat{d}_K}{\hat{c}_K}}}\right) - 1\right] \qquad (11)$$

as demonstrated by Huberman and Kandel (1987) and Jobson and Korkie (1989). The ratio given by $\left(\frac{\sqrt{\hat{c}_K}}{\sqrt{\hat{c}_{K+N}}}\right)$ is the ratio of the standard deviations of the minimum variance portfolios of the *K* benchmark assets and the *K+N* extended set of benchmark and test assets, bound below by 1. The ratio given by $\left(\frac{\sqrt{1+\frac{\hat{d}_{K+N}}{\hat{c}_{K+N}}}}{\sqrt{1+\frac{\hat{d}_K}{\hat{c}_K}}}\right)$ is the length of the asymptote to the *K+N* efficient frontier divided by the equivalent to the restricted frontier of the *K* benchmark assets, bound below by 1, as shown by Kan and Zhou (2008).

While Huberman and Kandel (1987) suggest testing the imposed constraints in (5) using the likelihood ratio (LR) test, Kan and Zhou (2008) demonstrate that the likelihood ratio (LR), Lagrange multiplier (LM), and Wald (W) tests are closely related spanning tests (see visualisation in Figure 7). Indeed, Kan and Zhou (2008) show that for the case when *N* = 1, the Wald test is the most powerful of the three tests.

Following this, we use Ordinary Least Squares estimation, and carry out a Wald test of the restrictions in (5).

Figure 7. Geometry of the MVS Tests



Notes: The geometry of the Wald (W), Lagrange Multiplier (LM), and Likelihood Ratio (LR) spanning tests, taken from Kan and Zhou (2008) and Berrill and Kearney (2010), are as below:

$$W = \left(\frac{OC}{OD}\right)^2 - 1 + \left(\frac{AE}{AF}\right)^2 - 1$$

$$LM = 1 - \left(\frac{OD}{OC}\right)^2 + 1 - \left(\frac{BG}{BH}\right)^2$$

$$LR = \left(\frac{OC}{OD}\right)\left(\frac{BH}{AF}\right) - 1$$